The PCX and DCX for the detector system

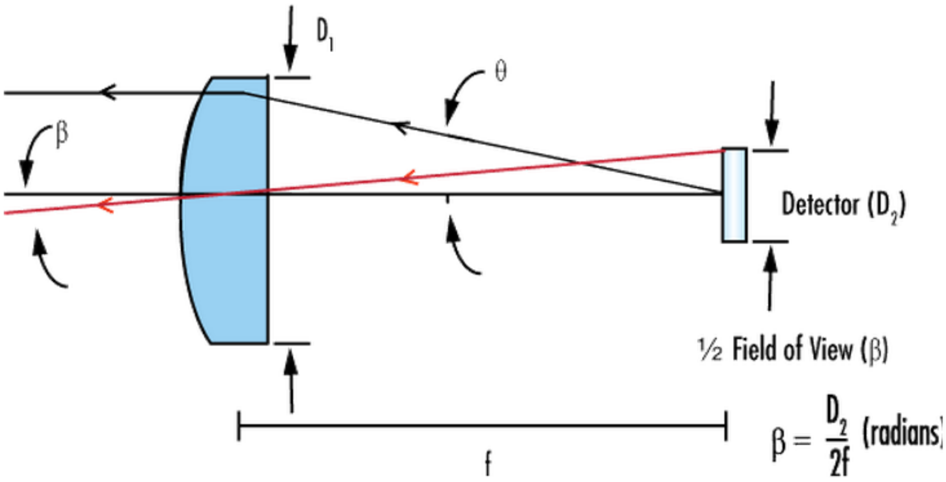

Figure 1: PCX Lens as FOV Limit in Detector Application

Every optical system requires some sort of preliminary design. Getting started with the design is often the most intimidating step, but identifying several important specifications of the system will help establish an initial plan. The following questions will illustrate the process of designing a simple detector or emitter system.

Goal: Where Will the Light Go?

Although simple lenses are often used in imaging applications, in many cases their goal is to project light from one point to another within a system. Nearly all emitters, detectors, lasers, and fiber optics require a lens for this type of light manipulation. Before determining which type of system to design, an important question to answer is "Where will the light go?" If the goal of the design is to get all incident light to fill a detector, with as few aberrations as possible, then a simple singlet lens, such as a plano-convex (PCX) lens or double-convex (DCX) lens, can be used.

Figure 1 shows a PCX lens, along with several important specifications: Diameter of the lens (D_1) and Focal Length (f). Figure 1 also illustrates how the diameter of the detector limits the Field of View (FOV) of the system, as shown by the approximation for Full Field of View (FFOV):

$$FFOV = \frac{D_2}{f} \tag{1.1}$$

Or, by the exact equation:

$$FFOV = 2\tan^{-1}\left(\frac{D_2}{2f}\right) \tag{1.2}$$

For detectors used in scanning systems, the important measure is the Instantaneous Field of View (IFOV), which is the angle subtended by the detector at any instant during scanning.

$$IFOV = \frac{Pixel\ Size}{f} \tag{1.3}$$

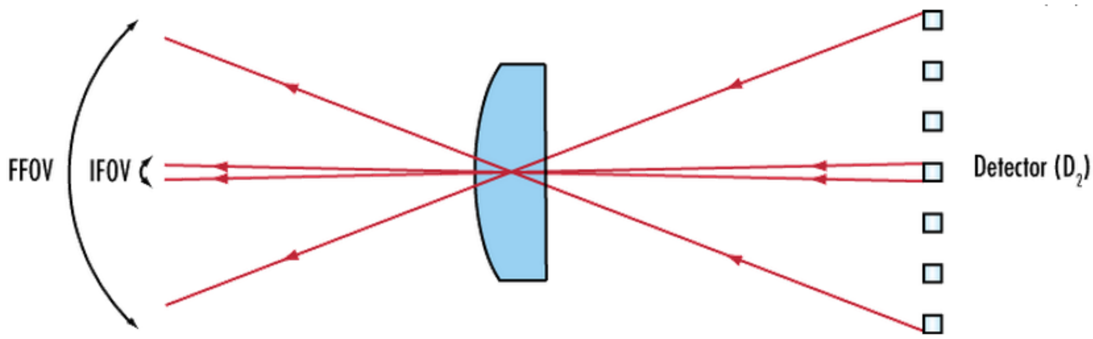

Figure 2: Instantaneous FOV

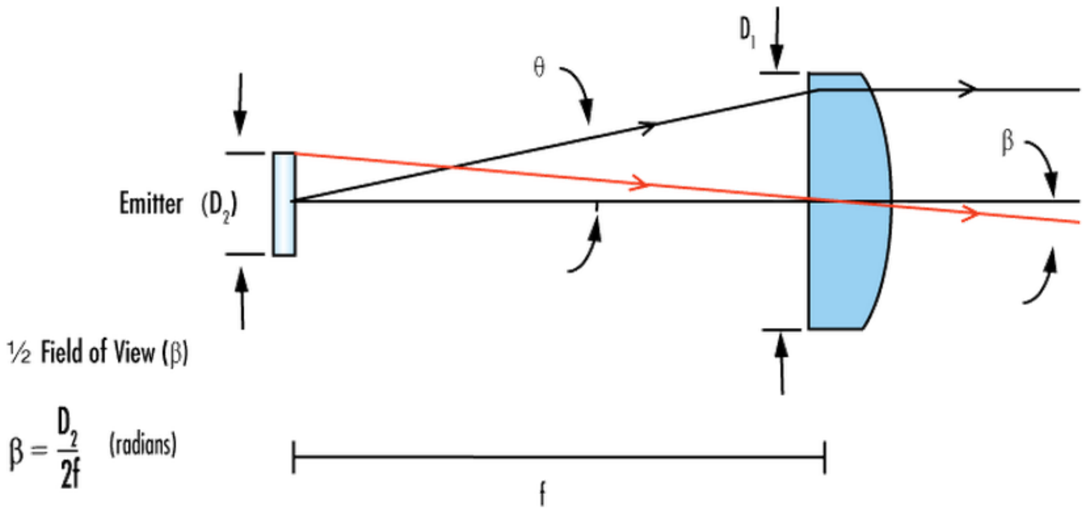

Figure 3: PCX Lens as FOV Limit in Emitter Application

Considered in reverse, Figure 1 can also represent an emitting system (Figure 3), with the lens used to collimate the light. This setup will be the premise of the application example.

Light Transmission: How Much Light Exists Initially?

Knowing where the light will go is only the first step in designing a light-projecting system; it is just as important to know how much light is transmitted from the object, or the source. The efficiency is based on how much light is received by the detector, thereby answering the question "How much light exists initially?" The Numerical Aperture (NA) and f-number (f/#) of a lens measure the amount of light it can collect based on f, D, index of refraction (n), and Acceptance Angle (θ). Figure 4 illustrates the relationship between f/# and NA.

Correspondingly, this relationship can be mathematically expressed according to Equation 1.5. It is important to note that the larger the Diameter, the smaller the f/#; this allows more light to enter the system. To create the most efficient system, it is best to match the emitted cone of light from the source to the acceptance cone of the lens, as this avoids over or under filling the lens area.

$$f\# = \frac{f}{D} \tag{1.4}$$

$$NA = \frac{1}{2(f\#)} \tag{1.5}$$

$$NA = n \sin\theta$$

$$\approx n\theta \tag{1.6}$$

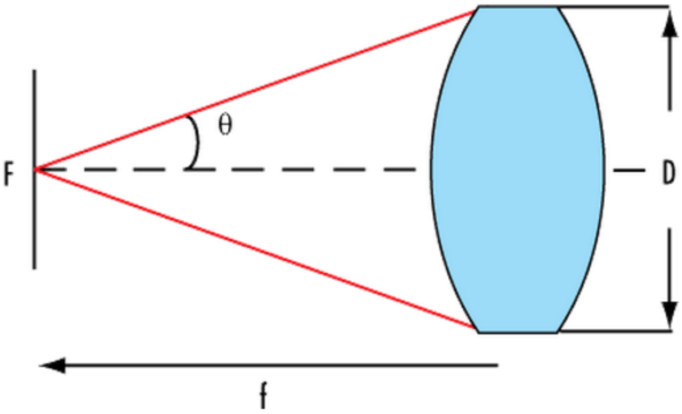

Figure 4: DCX Lens Showing f/# and NA

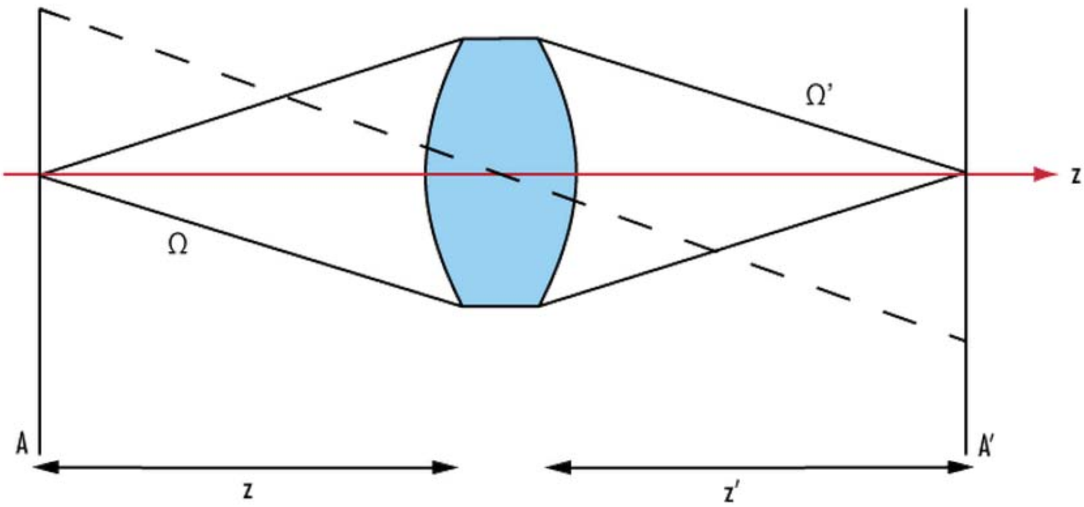

Figure 5: DCX Lens Illustrating Throughput

Optical Throughput: How Much Light gets through the System?

When using a lens as a tool to transfer light from an emitter to a detector, it is important to consider Throughput (TP), a quantitative measurement of transmitted light energy. In other words, answering the question "How much light gets through the system?" dictates the geometry of the lens used and the configuration of the system. Because emitters and detectors are areas of light and not point sources, the diameter of a lens affects TP even when the ratio between Diameter and Focal Length (specified by f/#) remains constant.

Figure 5 shows the basic definition of throughput (TP) as expressed in Equation 1.7, where A is the Area of the object, (light source), Ω is the Solid Angle, and z is the Object Distance (with their conjugates in image space as A', Ω', and z').

$$TP = n^2 A\Omega \tag{1.7}$$

Solid angle is defined as $\Omega = A/r^2$, with the area of the lens surface and the radius (r) being the distance from the lens to the object (z) or image plane (z'), for Ω or Ω', respectively.

The amount of light reaching the detector can be reduced by vignetting, which is the result of light being physically blocked within the system due to lens aperture limitations. However, some systems benefit from intentional vignetting, as it can eliminate stray light that would negatively affect the quality of the image. It is important to note that properly aligning the system reduces stray light and unintentional vignetting.

Aberrations: How Does the Image Look?

Determining how much light passes through the system is important, but aberrations within the system also play a major role. Answering "How does the image look?" can lead to improving the system's design in order to reduce aberrations and improve image quality. Aberrations are errors inherent with any optical system, regardless of fabrication or alignment. Since every optical system contains aberrations, balancing performance with cost is an important decision for any designer. Several basic aberrations, such as coma (variation in magnification or image size with aperture), spherical (light rays focusing in front of or behind paraxial focus), and astigmatism (having one focus point for horizontal rays and another for vertical) can be reduced by a large f/#, as shown in the following relations.

$$Spherical \propto \frac{1}{(f\#)^3} \tag{1.8}$$

$$Coma \propto \frac{1}{(f\#)^2} \tag{1.9}$$

$$Astigmatism \propto \frac{1}{(f\#)} \tag{1.10}$$

Application Example: Detector System

As an example, consider a system in which light is emitted from a ¼″ diameter fiber optic light guide, as shown in Figure 3.

- **Initial Parameters**

  NA of Light Guide = 0.55

  Diameter of Source (Emitter) = 6.35mm

  Index of Refraction of Air = 1
- **Calculated Parameters**

  F- Number (f/#)

$$NA = \frac{1}{2(f\#)}$$

$$0.55 = \frac{1}{2(f\#)}$$

$$f\# = 0.9 \tag{1.11}$$

A PCX lens of f/1, meaning the f/# is 1, would be ideal to place in front of the light guide in order to collimate as much light as possible. According to Equation 1.4, if the f/# is 1, then the diameter and focal length of a lens are equal. In other words, if we consider a lens with a diameter of 12mm, then the focal length is also 12mm.

Full Field of View (FFOV)

$$FFOV = \frac{D_2}{f}$$

$$FFOV = \frac{D_{source}}{f_{lens}}$$

$$= \frac{6.35mm}{12mm} = 0.529radians \tag{1.12}$$

Throughput (TP)

$$A_{Source} = \pi r^2$$

$$= \pi \left(\frac{6.35mm}{2}\right)^2 = 31.669mm^2 \tag{1.13}$$

$$A_{lens} = \pi r^2$$

$$= \pi \left(\frac{12mm}{2}\right)^2 = 113.097mm^2 \tag{1.14}$$

$$\Omega = \frac{Area_{lens}}{Radius^2}$$

$$= \frac{113.097mm^2}{(12mm)^2} = 0.7854steradians \tag{1.15}$$

Steradians correspond to a 2-dimensional angle in 3-dimensional space, as the angle from the edge to edge of the lens is in two dimensions. A higher value in steradians is given by a shorter distance from emitter to lens, or a larger diameter of the lens. The largest value a solid angle can have is 4π, or about 12.57, as this would be equivalent to the solid angle of all space.

In order to calculate Throughput (TP) of this system, we need to first calculate the Area of the Source (Equation 1.11), the Area of the Lens (Equation 1.12) and the Solid Angle (Equation 1.13). As a rule of thumb for collimating light from a divergent source (i.e. the light guide in this example), place the lens a distance equal to one focal length away from the source.

$$
\begin{aligned}
TP &= n^2 A\Omega \\
&= (1)(31.669mm^2)(0.7854steradians) \\
&= 24.873mm^2 steradians \qquad \textbf{(1.16)}
\end{aligned}
$$

Since the system is in free space, where n is approximated as 1, n^2 does not factor into the final calculation.